Moving in circles

The racing car in Figure 17.1 shows two examples of circular motion. The car's wheels spin around the axles, and the car follows a curved path as it speeds round the bend.

Figure 17.1 Circular motion: the car's wheels go round in circles as the car itself follows a curved path.

Describing circular motion

Many things move in circles. Here are some examples:

- the wheels of a car or a bicycle
- the Earth in its (approximately circular) orbit round the Sun
- the hands of a clock
- a spinning DVD in a laptop
- the drum of a washing machine.

Sometimes, things move along a path that is part of a circle. For example, the car in Figure 17.1 is travelling around a bend in the road which is an arc of a circle.

Circular motion is different from the straight-line motion that we have discussed previously in our study of kinematics and dynamics in Chapters 1–6. However, we can extend these ideas of dynamics to build up a picture of circular motion.

Around the clock

The second hand of a clock moves steadily round the clock face. It takes one minute for it to travel all the way round the circle. There are 360° in a complete circle and 60 seconds in a minute. So the hand moves 6° every second. If we know the angle θ through which the hand has moved from the vertical (12 o'clock) position, we can predict the position of the hand.

In the same way, we can describe the position of any object as it moves around a circle simply by stating the angle θ of the arc through which it has moved from its starting position. This is shown in Figure 17.2.

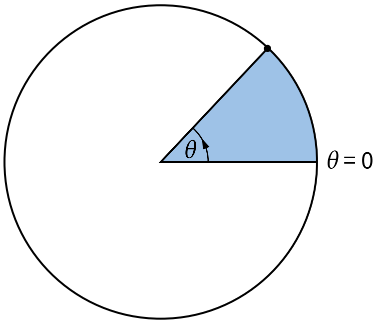

Figure 17.2 To know how far an object has moved round the circle, we need to know the angle θ.

The angle θ through which the object has moved is known as its **angular displacement**. For an object moving in a straight line, its position was defined by its displacement s, the **distance** it has travelled from its starting position. The corresponding quantity for circular motion is angular displacement θ, the **angle** of the arc through which the object has moved from its starting position.

QUESTION

1 **a** By how many degrees does the angular displacement of the hour hand of a clock change each hour?

b A clock is showing 3.30. Calculate the angular displacements in degrees from the 12.00 position of the clock to:

i the minute hand

ii the hour hand.

Angles in radians

When dealing with circles and circular motion, it is more convenient to measure angles and angular displacements in units called radians rather than in degrees. If an object moves a distance s around a circular path of radius r (Figure 17.3a), its angular displacement θ in **radians** is defined as follows:

$$\text{angle (in radians)} = \frac{\text{length of arc}}{\text{radius}}$$

or $$\theta = \frac{s}{r}$$

Since both s and r are distances measured in metres, it follows that the angle θ is simply a ratio. It is a dimensionless quantity. If the object moves twice as far around a circle of twice the radius (Figure 17.3b), its angular displacement θ will be the same.

$$\theta = \frac{\text{length of arc}}{\text{radius}} = \frac{2s}{2r} = \frac{s}{r}$$

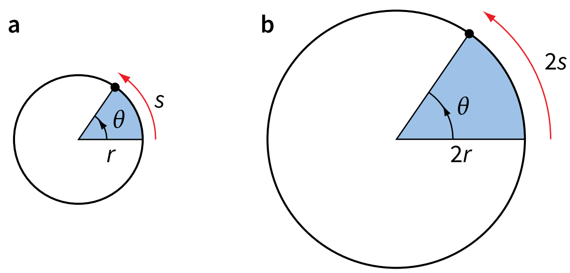

Figure 17.3 The size of an angle depends on the radius and the length of the arc. Doubling both leaves the angle unchanged.

When we define θ in this way, its units are radians rather than degrees. How are radians related to degrees? If an object moves all the way round the circumference of the circle, it moves a distance of $2\pi r$. We can calculate its angular displacement in radians:

$$\theta = \frac{\text{circumference}}{\text{radius}} = \frac{2\pi r}{r} = 2\pi$$

Hence a complete circle contains 2π radians. But we can also say that the object has moved through 360°. Hence:

$$360° = 2\pi \text{ rad}$$

Similarly, we have:

$$180° = \pi \text{ rad} \qquad 90° = \frac{\pi}{2} \text{ rad}$$

$$45° = \frac{\pi}{4} \text{ rad} \qquad \text{and so on}$$

Defining the radian

An angle of one radian is defined as follows (see Figure 17.4):

> One **radian** is the angle subtended at the centre of a circle by an arc of length equal to the radius of the circle.

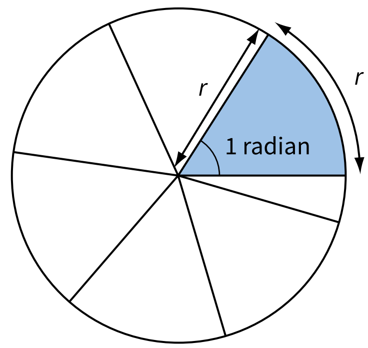

Figure 17.4 The length of the arc is equal to the radius when the angle is 1 radian.

An angle of 360° is equivalent to an angle of 2π radians. We can therefore determine what 1 radian is equivalent to in degrees.

$$1 \text{ radian} = \frac{360°}{2\pi}$$

or $$1 \text{ radian} \approx 57.3°$$

If you can remember that there are 2π rad in a full circle, you will be able to convert between radians and degrees:

- to convert from degrees to radians, multiply by $\frac{2\pi}{360°}$ or $\frac{\pi}{180°}$
- to convert from radians to degrees, multiply by $\frac{360°}{2\pi}$ or $\frac{180°}{\pi}$

Now look at Worked example 1.

WORKED EXAMPLE

1 If $\theta = 60°$, what is the value of θ in radians?

The angle θ is 60°. 360° is equivalent to 2π radians. Therefore:

$$\theta = 60 \times \frac{2\pi}{360}$$

$$= \frac{\pi}{3} = 1.05 \text{ rad}$$

(Note that it is often useful to express an angle as a multiple of π radians.)

QUESTION

2 a Convert the following angles from degrees into radians: 30°, 90°, 105°.

b Convert these angles from radians to degrees: 0.5 rad, 0.75 rad, π rad, $\frac{\pi}{2}$ rad.

c Express the following angles as multiples of π radians: 30°, 120°, 270°, 720°.

Steady speed, changing velocity

If we are to use Newton's laws of motion to explain circular motion, we must consider the **velocity** of an object going round in a circle, rather than its **speed**.

There is an important distinction between speed and velocity: speed is a scalar quantity which has magnitude only, whereas velocity is a vector quantity, with both magnitude and direction. We need to think about the direction of motion of an orbiting object.

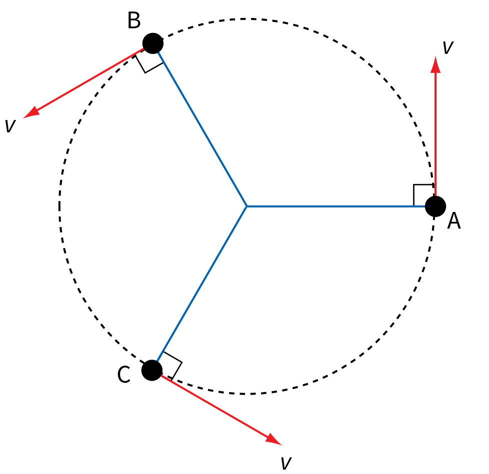

Figure 17.5 The velocity v of an object changes direction as it moves along a circular path.

Figure 17.5 shows how we can represent the velocity of an object at various points around its circular path. The arrows are straight and show the direction of motion at a particular instant. They are drawn as tangents to the circular path. As the object travels through points A, B, C, etc., its speed remains constant but its direction changes. Since the direction of the velocity v is changing, it follows that v itself (a vector quantity) is changing as the object moves in a circle.

Angular velocity

As the hands of a clock travel steadily around the clock face, their velocity is constantly changing. The minute hand travels round 360° or 2π radians in 3600 seconds. Although its velocity is changing, we can say that its angular velocity is constant, because it moves through the same angle each second:

$$\text{angular velocity} = \frac{\text{angular displacement}}{\text{time taken}}$$

$$\omega = \frac{\Delta\theta}{\Delta t}$$

We use the symbol ω (Greek letter omega) for angular velocity, measured in radians per second (rad s^{-1}). For the minute hand of a clock, we have $\omega = \frac{2\pi}{3600} \approx 0.001\,75$ rad s^{-1}.

QUESTIONS

5 Show that the angular velocity of the second hand of a clock is about 0.105 rad s^{-1}.

6 The drum of a washing machine spins at a rate of 1200 rpm (revolutions per minute).

a Determine the number of revolutions per second of the drum.

b Determine the angular velocity of the drum.

QUESTIONS

3 Explain why all the velocity arrows in Figure 17.5 are drawn the same length.

4 A toy train travels at a steady speed of 0.2 m s^{-1} around a circular track (Figure 17.6). A and B are two points diametrically opposite to one another on the track.

a Determine the change in the speed of the train as it travels from A to B.

b Determine the change in the velocity of the train as it travels from A to B.

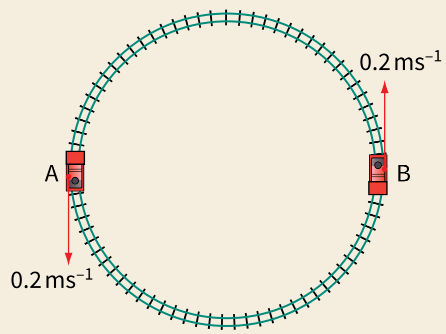

Figure 17.6 A toy train travelling around a circular track – for Question 4.

Relating velocity and angular velocity

Think again about the second hand of a clock. As it goes round, each point on the hand has the same angular velocity. However, different points on the hand have different velocities. The tip of the hand moves fastest; points closer to the centre of the clock face move more slowly.

This shows that the speed v of an object travelling around a circle depends on two quantities: its angular velocity ω and its distance from the centre of the circle r. We can write the relationship as an equation:

speed = angular velocity × radius

$$v = \omega r$$

Worked example 2 shows how to use this equation.

WORKED EXAMPLE

2 A toy train travels around a circular track of radius 2.5 m in a time of 40 s. What is its speed?

Step 1 Calculate the train's angular velocity ω. One circuit of the track is equivalent to 2π radians. The rain travels around in 10 s. Therefore:

$$\omega = \frac{2\pi}{40} = 0.157\,\text{rad}^{-1}$$

Step 2 Calculate the train's speed:

$$v = \omega r = 0.157 \times 2.5 = 0.39\,\text{m}\,\text{s}^{-1}$$

Hint: *You could have arrived at the same answer by calculating the distance travelled (the circumference of the circle) and dividing by the time taken.*

QUESTIONS

7 The angular velocity of the second hand of a clock is $0.105\,\text{rad}\,\text{s}^{-1}$. If the length of the hand is 1.8 cm, calculate the speed of the tip of the hand as it moves round.

8 A car travels around a 90° bend in 15 s. The radius of the bend is 50 m.

a Determine the angular velocity of the car.

b Determine the speed of the car.

9 A spacecraft orbits the Earth in a circular path of radius 7000 km at a speed of $7800\,\text{m}\,\text{s}^{-1}$. Determine its angular velocity.

Centripetal forces

When an object's velocity is changing, it has acceleration. In the case of uniform circular motion, the acceleration is rather unusual because, as we have seen, the object's speed does not change but its velocity does. How can an object accelerate and at the same time have a steady speed?

One way to understand this is to think about what Newton's laws of motion can tell us about this situation. **Newton's first law** states that an object remains at rest or in a state of uniform motion (at constant speed in a straight line) unless it is acted on by an external force. In the case of an object moving at steady speed in a circle, we have a body whose velocity is not constant; therefore, there must be a resultant (unbalanced) force acting on it.

Now we can think about different situations where objects are going round in a circle and try to find the force that is acting on them.

- Consider a rubber bung on the end of a string. Imagine whirling it in a horizontal circle above your head (Figure 17.7). To make it go round in a circle, you have to pull on the string. The pull of the string on the bung is the unbalanced force, which is constantly acting to change the bung's velocity as it orbits your head. If you let go of the string, suddenly there is no tension in the string and the bung will fly off at a tangent to the circle.
- Similarly, as the Earth orbits the Sun, it has a constantly changing velocity. Newton's first law suggests that there must be an unbalanced force acting on it. That force is the gravitational pull of the Sun. If the force disappeared, the Earth would travel off in a straight line.

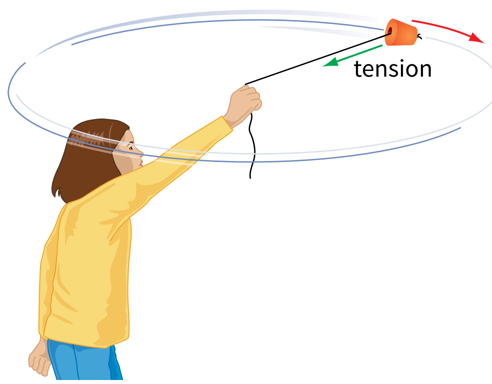

Figure 17.7 Whirling a rubber bung.

In both of these cases, you should be able to see why the direction of the force is as shown in Figure 17.8. The force on the object is directed towards the centre of the circle. We describe each of these forces as a **centripetal force** – that is, directed towards the centre.

It is important to note that the word **centripetal** is an adjective. We use it to describe a force that is making something travel along a circular path. It does not tell us what causes this force, which might be gravitational, electrostatic, magnetic, frictional or whatever.

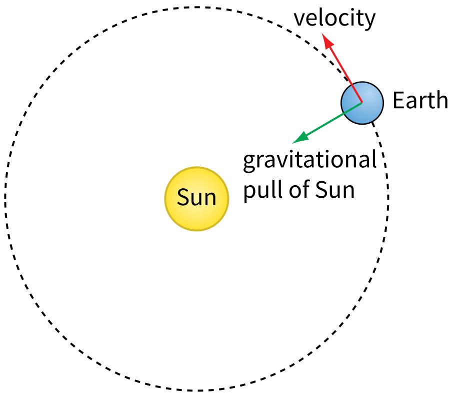

Figure 17.8 The gravitational pull of the Sun provides the centripetal force that keeps the Earth in its orbit.

QUESTIONS

10 In each of the following cases, state what provides the centripetal force:

- **a** the Moon orbiting the Earth
- **b** a car going round a bend on a flat, rough road
- **c** the weight on the end of a swinging pendulum.

11 A car is travelling along a flat road. Explain why it cannot go around a bend if the road surface is perfectly smooth. Suggest what might happen if the driver tries turning the steering wheel.

Vector diagrams

Figure 17.9a shows an object travelling along a circular path, at two positions in its orbit. It reaches position B a short time after A. How has its velocity changed between these two positions?

The change in the velocity of the object can be determined using a vector triangle. The vector triangle in Figure 17.9b shows the difference between the final velocity v_B and initial velocity v_A. The change in the velocity of the object between the points B and A is shown by the smaller arrow labelled Δv. Note that the change in the velocity of the object is (more or less):

- at right angles to the velocity at A
- directed towards the centre of the circle.

The object is accelerating because its velocity changes. Since acceleration is the rate of change of velocity:

$$a = \frac{\Delta v}{\Delta t}$$

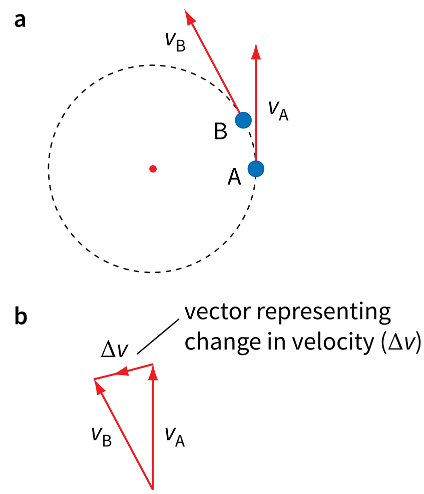

Figure 17.9 Changes in the velocity vector.

it follows that the acceleration of the object must be in the same direction as the change in the velocity – towards the centre of the circle. This is not surprising because, according to $F = ma$, the acceleration a of the object is in the same direction as the centripetal force F.

Acceleration at steady speed

Now that we know that the centripetal force F and acceleration are always at right angles to the object's velocity, we can explain why its speed remains constant. If the force is to make the object change its speed, it must have a component in the direction of the object's velocity; it must provide a push in the direction in which the object is already travelling. However, here we have a force at 90° to the velocity, so it has no component in the required direction. (Its component in the direction of the velocity is $F\cos 90° = 0$.) It acts to pull the object around the circle, without ever making it speed up or slow down.

You can also use the idea of work done to show that the speed of the object moving in a circle remains the same. The work done by a force is equal to the product of the force and the distance moved by the object in the direction of the force. The distance moved by the object in the direction of the centripetal force is zero; hence the work done is zero. If no work is done on the object, its kinetic energy must remain the same and hence its speed is unchanged.

QUESTION

12 An object follows a circular path at a steady speed. Describe how each of the following quantities changes as it follows this path: speed, velocity, kinetic energy, momentum, centripetal force, centripetal acceleration. (Refer to both magnitude and direction, as appropriate.)

Understanding circular motion

Isaac Newton devised an ingenious thought experiment that allows us to think about how an object can remain in a circular orbit around the Earth. Consider a large cannon on some high point on the Earth's surface, capable of firing objects horizontally. Figure 17.10 shows what will happen if we fire them at different speeds.

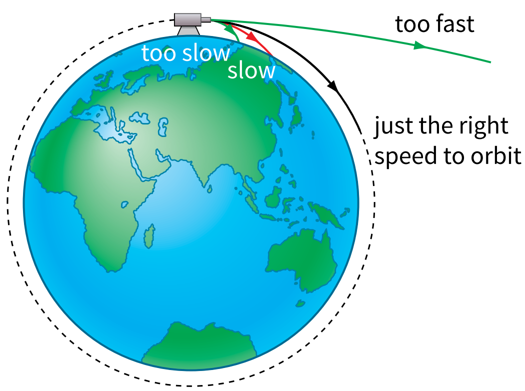

Figure 17.10 Newton's 'thought experiment'.

If the object is fired too slowly, gravity will pull it down towards the ground and it will land at some distance from the cannon. A faster initial speed results in the object landing further from the cannon.

Now, if we try a bit faster than this, the object will travel all the way round the Earth. We have to get just the right speed to do this. As the object is pulled down towards the Earth, the curved surface of the Earth falls away beneath it. The object follows a circular path, constantly falling under gravity but never getting any closer to the surface.

If the object is fired too fast, it travels off into space, and fails to get into a circular orbit. So we can see that there is just one correct speed to achieve a circular orbit under gravity. (Note that we have ignored the effects of air resistance in this discussion.)

Calculating acceleration and force

If we spin a bung around in a circle (Figure 17.7), we get a feeling for the factors which determine the centripetal force F required to keep it in its circular orbit. The greater the mass m of the bung and the greater its speed v, the greater is the force F that is required. However if the radius r of the circle is increased, F is smaller.

Now we will deduce an expression for the centripetal acceleration of an object moving around a circle with a constant speed. Figure 17.11 shows a particle moving round a circle. In time Δt it moves through an angle $\Delta\theta$ from A to B. Its speed remains constant but its velocity changes by Δv, as shown in the vector diagram. Since the narrow angle in this triangle is also $\Delta\theta$, we can say that:

$$\Delta\theta = \frac{\Delta v}{v}$$

Dividing both sides of this equation by Δt and rearranging gives:

$$\frac{\Delta v}{\Delta t} = \frac{v\Delta\theta}{\Delta t}$$

The quantity on the left is $\frac{\Delta v}{\Delta t} = a$, the particle's acceleration.

The quantity on the right is $\frac{\Delta\theta}{\Delta t} = \omega$, the angular velocity.

Substituting for these gives:

$$a = v\omega$$

Using $v = \omega r$, we can eliminate ω from this equation:

$$a = \frac{v^2}{r}$$

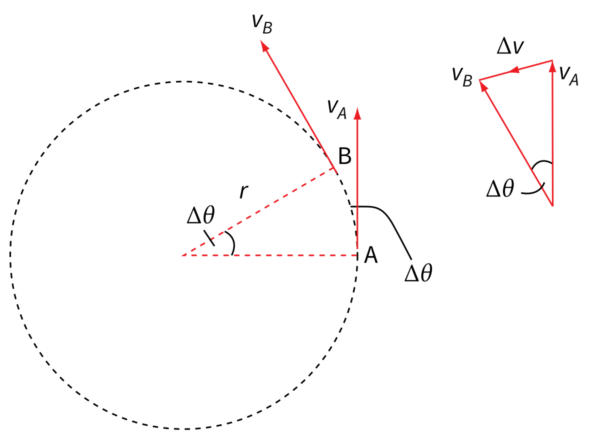

Figure 17.11 Deducing an expression for centripetal acceleration.

QUESTION

13 Show that an alternative equation for the centripetal acceleration is $a = \omega^2 r$.

Newton's second law of motion

Now that we have an equation for centripetal acceleration, we can use Newton's second law of motion to deduce an equation for centripetal force. If we write this law as $F = ma$, we find:

$$\text{centripetal force } F = \frac{mv^2}{r} = mr\omega^2$$

Remembering that an object accelerates in the direction of the resultant force on it, it follows that both F and a are in the same direction, towards the centre of the circle.

Calculating orbital speed

We can use the force equation to calculate the speed that an object must have to orbit the Earth under gravity, as in Newton's thought experiment. The necessary centripetal force $\frac{mv^2}{r}$ is provided by the Earth's gravitational pull mg. Hence:

$$mg = \frac{mv^2}{r}$$

$$g = \frac{v^2}{r}$$

where $g = 9.81\ \text{m s}^{-2}$ is the acceleration of free fall close to the Earth's surface. The radius of its orbit is equal to the Earth's radius, approximately 6400 km. Hence, we have:

$$9.81 = \sqrt{\frac{v^2}{(6.4 \times 10^6)}}$$

$$v = \sqrt{9.81 \times 6.4 \times 10^6} \approx 7.92 \times 10^3\ \text{m s}^{-1}$$

Thus if you were to throw or hit a ball horizontally at almost $8\ \text{km s}^{-1}$, it would go into orbit around the Earth.

The origins of centripetal forces

It is useful to look at one or two situations where the physical origin of the centripetal force may not be immediately obvious. In each case, you will notice that the forces acting on the moving object are not balanced – there is a resultant force. An object moving along a circular path is not in equilibrium and the resultant force acting on it is the centripetal force.

1 A car cornering on a level road (Figure 17.13). Here, the road provides two forces. The force N is the normal contact force which balances the weight mg of the car – the car has no acceleration in the vertical direction.

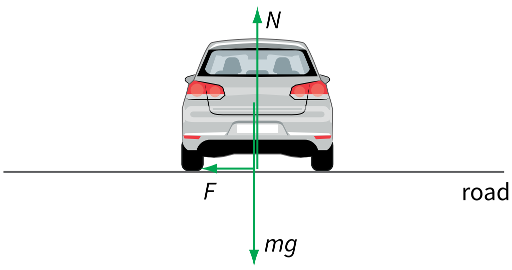

Figure 17.13 This car is moving away from us and turning to the left. Friction provides the centripetal force. N and F are the total normal contact and friction forces (respectively) provided by the contact of all four tyres with the road.

QUESTIONS

14 Calculate how long it would take a ball to orbit the Earth once, just above the surface, at a speed of $7920\ \text{m s}^{-1}$. (The radius of the Earth is 6400 km.)

15 A stone of mass 0.20 kg is whirled round on the end of a string of length 30 cm. The string will break when the tension in it exceeds 8.0 N. Calculate the maximum speed at which the stone can be whirled without the string breaking.

Figure 17.12 The view from the International Space Station, orbiting Earth over Australia.

16 The International Space Station (Figure 17.12) has a mass of 350 tonnes, and orbits the Earth at an average height of 340 km, where the gravitational acceleration is $8.8\ \text{m s}^{-2}$. The radius of the Earth is 6400 km. Calculate:

- **a** the centripetal force on the space station
- **b** the speed at which it orbits
- **c** the time taken for each orbit
- **d** the number of times it orbits the Earth each day.

17 A stone of mass 0.40 kg is whirled round on the end of a string 0.50 m long. It makes three complete revolutions each second. Calculate:

- **a** its speed
- **b** its centripetal acceleration
- **c** the tension in the string.

18 Mars orbits the Sun once every 687 days at a distance of 2.3×10^{11} m. The mass of Mars is 6.4×10^{23} kg. Calculate:

- **a** its orbital speed
- **b** its centripetal acceleration
- **c** the gravitational force exerted on Mars by the Sun.

The second force is the force of friction F between the tyres and the road surface. This is the unbalanced, centripetal force. If the road or tyres do not provide enough friction, the car will not go round the bend along the desired path. The friction between the tyres and the road provides the centripetal force necessary for the car's circular motion.

2 A car cornering on a banked road (Figure 17.14a). Here, the normal contact force N has a horizontal component which can provide the centripetal force. The vertical component of N balances the car's weight. Therefore:

vertically $\quad N\cos\theta = mg$

horizontally $\quad N\sin\theta = \dfrac{mv^2}{r}$

where r is the radius of the circular corner and v is the car's speed.

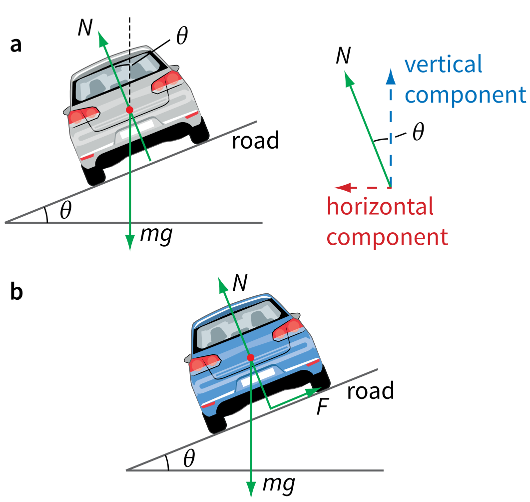

Figure 17.14 **a** On a banked road, the horizontal component of the normal contact force from the road can provide the centripetal force needed for cornering. **b** For a slow car, friction acts up the slope to stop it from sliding down.

If a car travels around the bend too slowly, it will tend to slide down the slope and friction will act up the slope to keep it on course (Figure 17.14b). If it travels too fast, it will tend to slide up the slope. If friction is insufficient, it will move up the slope and come off the road.

3 An aircraft banking (Figure 17.15a). To change direction, the pilot tips the aircraft's wings. The vertical component of the lift force L on the wings balances the weight. The horizontal component of L provides the centripetal force.

4 A stone being whirled in a horizontal circle on the end of a string – this arrangement is known as a conical pendulum (Figure 17.15b). The vertical component of the tension T is equal to the weight of the stone. The horizontal component of the tension provides the centripetal force for the circular motion.

5 At the fairground (Figure 17.15c). As the cylinder spins, the floor drops away. Friction balances your weight. The normal contact force of the wall provides the centripetal force. You feel as though you are being pushed back against the wall; what you are feeling is the push of the wall on your back.

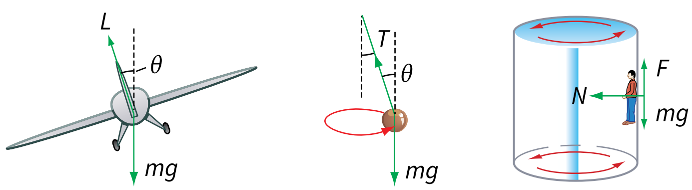

Figure 17.15 Three more ways of providing a centripetal force.

Note that the three situations shown in Figure 17.14a, Figure 17.15a and Figure 17.15b are equivalent. The moving object's weight acts downwards. The second force has a vertical component, which balances the weight, and a horizontal component, which provides the centripetal force.

QUESTIONS

19 Explain why it is impossible to whirl a bung around on the end of a string in such a way that the string remains perfectly horizontal.

20 Explain why an aircraft will tend to lose height when banking, unless the pilot increases its speed to provide more lift.

21 If you have ever been down a water-slide (a flume) (Figure 17.16) you will know that you tend to slide up the side as you go around a bend. Explain how this provides the centripetal force needed to push you around the bend. Explain why you slide higher if you are going faster.

Figure 17.16 A water-slide is a good place to experience centripetal forces.

Summary

- Angles can be measured in radians. An angle of 2π rad is equal to 360°.
- An object moving at a steady speed along a circular path has uniform circular motion.
- The angular displacement θ is a measure of the angle through which an object moves in a circle.
- The angular velocity ω is the rate at which the angular displacement changes: $\omega = \frac{\Delta\theta}{\Delta t}$
- For an object moving with uniform circular motion, speed and angular velocity are related by $v = \omega r$.
- An object moving in a circle is not in equilibrium; it has a resultant force acting on it.
- The resultant force acting on an object moving in a circle is called the centripetal force. This force is directed towards the centre of the circle and is at right angles to the velocity of the object.
- An object moving in a circle has a centripetal acceleration a given by:

  $$a = \frac{v^2}{r} = r\omega^2$$

- The magnitude of the centripetal force F acting on an object of mass m moving at a speed v in a circle of radius r is given by:

  $$F = \frac{mv^2}{r} = mr\omega^2$$

End-of-chapter questions

1 **a** Explain what is meant by a **radian**. [1]

b A body moves round a circle at a constant speed and completes one revolution in 15 s. Calculate the angular velocity of the body. [2]

2 Figure 17.17 shows part of the track of a roller-coaster ride in which a truck loops the loop. When the truck is at the position shown, there is no reaction force between the wheels of the truck and the track. The diameter of the loop in the track is 8.0 m.

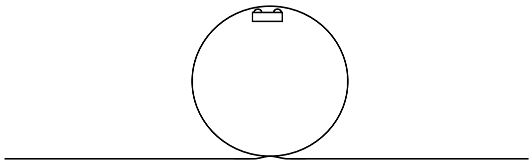

Figure 17.17 For End-of-chapter Question 2.

a Explain what provides the centripetal force to keep the truck moving in a circle. [1]

b Given that the acceleration due to gravity g is 9.8 m s^{-2}, calculate the speed of the truck. [3]

3 **a** Describe what is meant by **centripetal force**. [1]

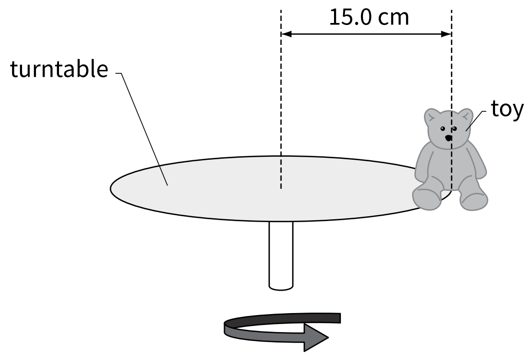

Figure 17.18 For End-of-chapter Question 3.

b Figure 17.18 shows a toy of mass 60 g placed on the edge of a rotating turntable.

i The diameter of the turntable is 15.0 cm. The turntable rotates, making 20 revolutions every minute. Calculate the centripetal force acting on the toy. [4]

ii Explain why the toy falls off when the speed of the turntable is increased. [2]

4 One end of a string is secured to the ceiling and a metal ball of mass 50 g is tied to its other end. The ball is initially at rest in the vertical position. The ball is raised through a vertical height of 70 cm (see Figure 17.19). The ball is then released. It describes a circular arc as it passes through the vertical position.

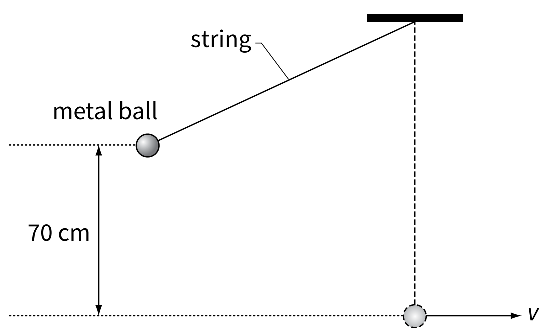

Figure 17.19 For End-of-chapter Question 4.

The length of the string is 1.50 m.

a Ignoring the effects of air resistance, determine the speed *v* of the ball as it passes through the vertical position. [2]

b Calculate the tension *T* in the string when the string is vertical. [4]

c Explain why your answer to **b** is not equal to the weight of the ball. [2]

5 A car is travelling round a bend when it hits a patch of oil. The car slides off the road onto the grass verge. Explain, using your understanding of circular motion, why the car came off the road. [2]

6 Figure 17.20 shows an aeroplane banking to make a horizontal turn. The aeroplane is travelling at a speed of $75\,m\,s^{-1}$ and the radius of the turning circle is 80 m.

a Copy the diagram. On your copy, draw and label the forces acting on the aeroplane. [2]

b Calculate the angle which the aeroplane makes with the horizontal. [4]

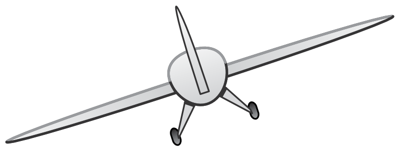

Figure 17.20 For End-of-chapter Question 6.

7 **a** Explain what is meant by the term **angular velocity**. [2]

b Figure 17.21 shows a rubber bung, of mass 200 g, on the end of a length of string being swung in a horizontal circle of radius 40 cm. The string makes an angle of 56° with the vertical.

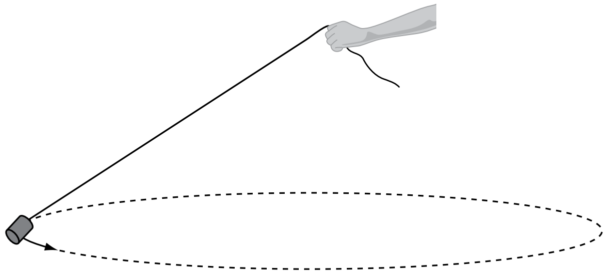

Figure 17.21 For End-of-chapter Question 7.

Calculate:

i the tension in the string [2]

ii the angular velocity of the bung [3]

iii the time it takes to make one complete revolution. [1]

8 **a** Explain what is meant by a **centripetal force**. [2]

b A teacher swings a bucket of water, of total mass 5.4 kg, round in a vertical circle of diameter 1.8 m.

i Calculate the minimum speed which the bucket must be swung at so that the water remains in the bucket at the top of the circle. [3]

ii Assuming that the speed remains constant, what will be the force on the teacher's hand when the bucket is at the bottom of the circle? [2]

9 In training, military pilots are given various tests. One test puts them in a seat on the end of a large arm which is then spun round at a high speed, as shown in Figure 17.22.

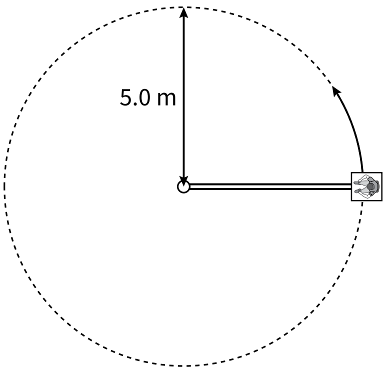

Figure 17.22 For End-of-chapter Question 9.

a Describe what the pilot will feel and relate this to the centripetal force. [3]

b At top speed the pilot will experience a centripetal force equivalent to six times his own weight (6 mg).

i Calculate the speed of the pilot in this test. [3]

ii Calculate the number of revolutions of the pilot per minute. [2]

c Suggest why it is necessary for pilots to be able to be able to withstand forces of this type. [2]

10 **a** Show that in one revolution there are 2π radians. [2]

b Figure 17.23 shows a centrifuge used to separate solid particles suspended in a liquid of lower density. The container is spun at a rate of 540 revolutions per minute.

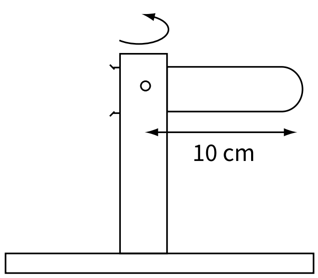

Figure 17.23 For End-of-chapter Question 10.

i Calculate the angular velocity of the container. [2]

ii Calculate the centripetal force on a particle of mass 20 mg at the end of the test tube. [2]

c An alternative method of separating the particles from the liquid is to allow them to settle to the bottom of a stationary container under gravity.

By comparing the forces involved, explain why the centrifuge is a more effective method of separating the mixture. [2]